Section 3.3 - Rules for Differentiation

Rule 1

Derivative of a Constant Function

If f is the function with the constant value c, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

Proof of Rule 1

$f(x) = x^3$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \lim_{h \to 0} \frac{(x+h)(x+h)(x+h) - x^3}{h}$$

$$= \lim_{h \to 0} \frac{(x^2 + 2xh + h^2)(x+h) - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 2x^2h + h^2x + x^2h + 2xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \to 0} (3x^2 + 3xh + h^2)$$

$$= 3x^2$$

$$f(x) = x^2$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} (2x + h) = 2x$$

$$f(x) = x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h} = \lim_{h \to 0} \frac{h}{h} = 1$$

Find the derivative of $f(x) = x^3$

Rule 2

Power Rule ~~for Positive Integer Powers of x~~

Or just the POWER RULE

~~If n is a positive integer~~, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Find the derivate of $4x^2$

$8x$

a) $f(x) = x^5$

$f'(x) = 5x^4$

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[5]{x} = x^{\frac{1}{5}}$

b) $g(x) = \sqrt[5]{x}$

$g(x) = x^{\frac{1}{5}}$

$g'(x) = \frac{1}{5}x^{-\frac{4}{5}}$

$= \frac{1}{5x^{\frac{4}{5}}}$

$= \frac{1}{5\sqrt[5]{x^4}}$

c) $h(x) = \frac{1}{x^5}$

$h'(x) = -5x^{-6}$

$= \frac{-5}{x^6}$

$h(x) = x^{-5}$

$h'(x) = -5x^{-6}$

Rule 3

The Constant Multiple Rule

$y = 4x^2$ $\qquad y' = 4 * 2x = 8x$

If u is a differentiable function of x and c, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}$$

In ENGLISH

- If a differentiable function is multiplied by a constant, then its derivative is multiplied by the same constant

a) $y = \frac{3}{\sqrt[4]{x}} = \frac{3}{x^{\frac{1}{4}}}$

$y = 3 \cdot x^{-\frac{1}{4}}$

$y' = \frac{-3}{4} x^{-\frac{5}{4}}$ ←

$= \frac{-3}{4x^{\frac{5}{4}}}$

$= \frac{-3}{4\sqrt[4]{x^5}}$

b) $y = \frac{3}{2x^2}$

$y = \frac{3x^{-2}}{2}$

$y' = \frac{-6x^{-3}}{2}$

$= -3x^{-3}$

$= \frac{-3}{x^3}$